MICROCOPY RESOLUTION TEST CHART

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AN EFFICIENT PATCHED GRID NAVIER-STOKES SOLUTION FOR MULTIPLE BODIES

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PHASE I
FINAL REPORT: R88-930015-F

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U.S. ARMY RESEARCH OFFICE
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U.S. DEPARTMENT OF DEFENSE

SMALL BUSINESS INNOVATION RESEARCH (SBIR) PROGRAM
PHASE 1—FY 1987
PROJECT SUMMARY

Topic No. __A87-115__

Military Department/Agency __Army__ .

Name and Address of Proposing Small Business Firm

> Scientific Research Associates, Inc.
> P.O. Box 1058
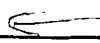> Glastonbury, CT 06033

Name and Title of Principal Investigator

> Bernard C. Weinberg, Senior Research Scientist

Proposal Title

> An Efficient Patched Grid Navier-Stokes Solution Procedure for Multiple Bodies

Technical Abstract (Limit your abstract to 200 words with no classified or proprietary information/data.)

A major problem area in current computational fluid dynamics technology concerns flows about complex configurations formed by multiple components in relative motion. Major difficulties encountered in such problems are those associated with the grid. For such applications, the geometric constraints of the component elements often require that patched grids be employed. Herein, a novel and efficient procedure is described to solve the time-dependent, multidimensional Navier-Stokes equations about multiple body configurations. In contrast to existing patched grid approaches, the present method calculates the entire flow field over both grids simultaneously, without iteration. By eliminating iteration within a time step and allowing time steps to be chosen by accuracy considerations, rather than by stability limits, this procedure could lead to a substantial savings in computer run time. In addition, for steady state problems improved convergence rates could be expected. To demonstrate the capabilities and advantages of the new procedure, a problem of current interest in turbomachinery, the flow field in a rotor-stator stage, is investigated using the developed procedure. A steady state flow field about a cascade of displaced tandem Joukowski airfoils is considered. The accuracy of the calculations and CPU time used are compared with a calculation using a continuous deformed grid algorithm and a patched grid with iteration.

Anticipated Benefits/Potential Commercial Applications of the Research or Development

The results of the Phase I effort indicate that the noniterative patched grid procedure would lead to a versatile and efficient Navier-Stokes computer code which would have a major cost effectiveness advantage over current patched grid procedures. Although demonstrated on a model rotor-stator problem the procedure would have much wider application including the fuselage-rotor problem, the impeller-diffuser problem, computation of flow over missiles with fins, etc. Potential commercial applications include consulting services, training and support of industrial clients and leasing of the code through a commercial computer group.

List a maximum of 8 Key Words that describe the Project.

Patched Grid, Noniterative, Navier-Stokes, Rotor-Stator

TABLE OF CONTENTS

INTRODUCTION

A major problem area in current computational fluid dynamics technology concerns flows about complex configurations formed by multiple components which may be in relative motion. Typical examples for multiple component configurations in which the components are in relative motion include the rotor-stator configuration in an axial compressor or turbine, the compressor-diffuser combination in a radial turbomachine, the rotor blade-fuselage and tail rotor-fuselage combinations found in a helicopter, etc. Important cases in which the components are not in relative motion include the wing-fuselage problem, the missile-fin problem and the wing-store problem. In all these cases, it may be possible to utilize a single grid for the flow field of interest, however, a single grid having sufficient resolution, smoothness and orthogonality properties may be difficult, if not impossible to construct. Hence, there has been a great deal of interest in developing technology for patched or overlaid grids (e.g., Refs. 1 and 2)

Clearly patched or overlaid grids contain significant advantages over a single grid in their ability to obtain necessary resolution while not unduly sacrificing orthogonality and smoothness requirements. However, patched and overlaid grids present their own problems. In particular, the internal boundaries between different patches presents a numerical problem and the question arises as to how to treat the patch line or the overlay region without unduly sacrificing computational efficiency. In general, the patched grid approach requires each region to be solved separately with the solution for the entire domain obtained by an iterative procedure between patches. In such a technique, the boundary conditions on the patched line for a given patch region are obtained from the latest solution for the adjoining regions.

Although patched and overlaid approaches do allow solutions for configurations which would be difficult if not impossible to analyze with a single grid approach, the iterative treatment of the patched line or overlaid region leads to a serious loss in computational efficiency. It is currently generally agreed that implicit treatment of boundary conditions is required to obtain rapid convergence for Navier-Stokes solvers. For example, as shown in Refs. 3 and 4, converged Navier-Stokes solutions, to engineering accuracy, for two- and three-dimensional, high Reynolds number compressor and turbine cascade

flow fields were obtained in 100 to 200 time steps. Explicit treatment of boundary conditions or patched grid conditions are expected to increase the required number of iterations (or time steps) to convergence by a factor of five to fifty. One possible solution to minimize this slow convergence rate would be to iterate on the patch boundary at each time or relaxation step. However, this effectively adds additional time steps to convergence. Therefore, the explicit treatment of the patch boundary decreases the convergence rate of the solution to steady state, thus increasing run time, turnaround time and cost.

A similar problem arises with unsteady flows in which the patched regions move one relative to another, as in a rotor-stator stage simulation of a rotorcraft blade-fuselage simulation. If an explicit boundary condition is used on the patch line, the maximum allowable time step will be governed by both physical considerations, i.e., the relevant physical time scale and numerical considerations in treating the patch boundaries explicitly. In general, the latter constraint is expected to dominate and, therefore, in such unsteady simulations the maximum allowable time step and consequently the required run time may be governed by the explicit patch boundary condition formulation. Clearly, an alternate procedure obtaining the advantages of both patched and single grid approaches would be an important and desirable innovation.

In the present effort an efficient Navier-Stokes analysis was developed for steady or time-dependent flows for multi-body configurations in which noniterative patched grids are utilized, thus combining the best features of single and patched grid approaches. The particular study used the turbomachine rotor-stator configuration as a test example. The objective was to demonstrate the practicality of and determine the efficiency of the innovative procedure for solving the Navier-Stokes equations on a patched grid. The efficiency of the method can be measured by comparison of the computational effort required by the noniterative patched grid approach with the efforts required by both the usual iterative patched grid approach and the single grid approach. The objectives were met by comparing the noniterative patched grid solution for a heat conduction problem with the single grid solution and the analytic solution. The practicality of the procedure for fluid mechanics problems was demonstrated by calculation for flow through two adjacent blade rows of Joukowski airfoils in which one row is displaced one-half gap relative to the other.

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PHASE I TECHNICAL OBJECTIVES

The overall goal of the present effort was to develop an efficient
Navier-Stokes analysis for steady or time-dependent flows for multi-body
configurations in which patched or overlaid grids are utilized. This
particular study employed a rotor-stator configuration as a test example. The
Phase I technical objective was to determine the efficiency of the numerical
procedure for solving the Navier-Stokes equations on a patched grid. The
individual objectives were:

1) to determine the additional computational effort required to solve the
   governing equations noniteratively,

2) to determine the relative efficiency of the numerical method with
   respect to the iterative approach, and

3) to determine the accuracy of the approach relative to a single
   non-patched grid.

GOVERNING EQUATIONS

The equations used are the ensemble-averaged, time-dependent
Navier-Stokes equations which can be written in vector form as

Continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{U} = 0 \tag{1}$$

Momentum

$$\frac{\partial \rho \vec{U}}{\partial t} + \nabla \cdot (\rho \vec{U} \vec{U}) = -\nabla P + \nabla \cdot \overline{\overline{\pi}} \tag{2}$$

where ρ is density, $\vec{U}$ is velocity, p is pressure $\overline{\overline{\pi}}$ is the molecular stress
tensor. The stress tensor appearing in Eq. (7) is defined as

$$\overline{\overline{\pi}} = 2\mu D - \left(\frac{2}{3}\mu - K_B\right)\nabla \cdot \vec{U} I \tag{3}$$

where K_B is the bulk viscosity coefficient, I is the identity tensor, and
is the deformation tensor, defined by:

$$D = \frac{1}{2} ((\nabla \vec{U}) + (\nabla \vec{U})^T) \tag{4}$$

The equation of state for a perfect gas is

$$P = \rho RT \tag{5}$$

where R is the gas constant and supplements the equations of motion.

The governing equations, Eqs. (1)-(5), are written in general vector form
and prior to their application to specific problems, it is necessary to decide
upon both a set of dependent variables and a proper coordinate transformation.
Based upon previous investigations (e.g. Ref. 4) the specific scalar momentum
equations to be solved are the x, y and z Cartesian momentum equations. The
dependent variables chosen are the physical Cartesian velocities u, v, w and
the density ρ. If an energy equation is included, then enthalpy would be added
to the set of dependent variables. In this paper, since two-dimensional
problems are considered, it is required to solve the x and y momentum equations
with u, v and ρ as the dependent variables.

The equations are then transformed to a general coordinate system in which
the general coordinates, y^j are related to the Cartesian coordinates, x_1 and
x_2 by

$$y^j = y^j(x_1, x_2, t); \quad j = 1, 2$$
$$\tau = t \tag{6}$$

As implied by Eq. 6, the general coordinate y^j may be a function of both the
Cartesian coordinates and time. This coordinate time dependence will have an
implication in so far as the choice of governing equation form is concerned.

The governing equations can be expressed in terms of the new independent
variables y^j as

$$\frac{\partial W}{\partial \tau} + \xi_t \frac{\partial W}{\partial \xi} + \xi_x \frac{\partial F}{\partial \xi} + \xi_y \frac{\partial G}{\partial \xi} + \eta_t \frac{\partial W}{\partial \eta} + \eta_x \frac{\partial F}{\partial \eta} + \eta_y \frac{\partial G}{\partial \eta}$$

$$= \frac{1}{Re} \left(\xi_x \frac{\partial F_1}{\partial \xi} + \eta_x \frac{\partial F_1}{\partial \eta} + \xi_y \frac{\partial G_1}{\partial \xi} + \eta_y \frac{\partial G_1}{\partial \eta} \right)$$

(7)

through a straight forward application of chain rule differentiation. In Eq. 7

$$\xi = y^1$$
$$\eta = y^2$$

and

$$W = \begin{bmatrix} \rho \\ \rho u \\ \rho v \end{bmatrix} \qquad F = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \end{bmatrix} \qquad G = \begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \end{bmatrix}$$

$$F_1 = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \end{bmatrix} \qquad G_1 = \begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \end{bmatrix}$$

NUMERICAL PROCEDURE

Prior to discussing the specific configurations which were computed under the present study, it is appropriate to discuss the numerical procedure used to solve the patched region domain. The basic procedure used is the Linearized Block Implicit (LBI) procedure of Briley and McDonald (Refs. 5 and 6).

The LBI method can be outlined as follows: the governing equations are replaced by an implicit time difference approximation, optionally a backward difference or Crank-Nicolson scheme. Terms involving nonlinearities at the implicit time level are linearized by Taylor series expansion about the solution at known time level, and spatial difference approximations are introduced. The result is a system of multidimensional coupled (but linear) difference equations for the dependent variables at the unknown or implicit time level. To solve these difference equations, the Douglas-Gunn procedure for generating alternating-direction implicit (ADI) splitting schemes as perturbations of fundamental implicit difference schemes is introduced in its natural extension to systems of partial differential equations. This ADI

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splitting technique leads to systems of coupled linear difference equations having narrow block-banded matrix structures which can be solved efficiently by standard block-elimination methods.

In the present study, an interpolated procedure for treating the noniterative patched lines was developed and incorporated within the LBI code. Figure 1 shows the schematic of the patched grid. For simplicity, uniform grids are shown for both grid systems, however, the algorithm is by no means restricted to uniform grids and, as will be shown, calculations were run for highly non-uniform grids. For each grid system a patched line was introduced at the interface. In the LBI procedure a solution is obtained by conducting two separate sweeps. For the grid shown in Figure 1, the sweep where vertical lines are treated implicitly contains no complication due to the patch. The horizontal sweep contains lines which are discontinuous, and it is for this sweep that the standard solution procedure would solve the left hand region with an explicit boundary condition specified on the patch. This explicit boundary condition would be obtained from the latest solution of the right hand region. The horizontal sweep for the right hand region would then be solved by assuming an explicit boundary condition on the patch from the latest solution for the left hand region.

The present procedure utilizes a different approach which allows a single set of implicit horizontal sweeps which cover both regions. Considering the left hand region first, the equations can be solved for the left hand region points on the patch line by introducing an additional line as shown. Since this additional line introduces an additional set of grid points, additional equations are required. These equations are obtained from a geometric interpolation formula from the other grid system. Similarly, an additional set of points aligned with the right hand side region is introduced by the left hand region. This allows the finite difference equations to be constructed for the patched line points belonging to the right hand region. Again, the additional points introduced require additional equations which are obtained from geometric interpolation of the points in the left hand region.

The noniterative patched algorithm follows along the same lines as the standard ADI procedure for a single deforming coordinate system. In each sweep the discretized equations at each grid point in a line are written out. This forms a partitioned block diagonal matrix in which each block is independent and is composed of a block tridiagonal matrix corresponding to each line in the

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sweep direction. However, for a patched grid, the partitioned blocks are not independent, since they introduce additional, as yet unknown, dependent variables, and thus cannot be solved separately. To uniquely specify these unknown values, additional equations are required. These are obtained from geometric interpolation formulas for the variables on the corresponding grids. The form of the matrices to be considered is shown in Figure 2. Consider first submatrix A, which was the upper left hand corner submatrix. This represents the algebraic set of equations governing the upstream grid, i.e. the grid on the left, for the first sweep of an ADI procedure. It should be emphasized that this submatrix contains all lines in the sweep. An individual line is contained within the noted sub-block within A. Similarly, submatrix D represents all the lines in the downstream or grid on the right. The matching conditions are contained within the lower four submatrices, which include two banded matrices, the band width of which depends on the order of the interpolation polynomial and the relative uniformity of the two grids on the patched location.

With some matrix manipulation, the system of equations can be reduced to a form in which the "left" and "right" interpolation matrices become decoupled and hence solved independently. The form of the matrix is shown in Figure 3. Although the "left" and "right" sides are decoupled, in reality the respective blocks on the left and the right have been employed to reduce the matrix. At this point, the two "decoupled" sub-blocks, can be inverted independently and the resultant values could then be used to solve for the unknowns in the remaining partitioned blocks. The problem has thus been reduced to a sequence of matrix inversion problems which are amenable to vectorization.

CALCULATIONS

Test Case 1 – Heat Conduction Problem

In order to validate the innovative procedure, several test calculations have been carried out. The first test case was to apply the technique to a two-dimensional scalar heat conduction equation in a patched grid configuration. Figure 4 shows the coordinate system of the patched grid with uniform spacing. Function boundary conditions were applied on the top and bottom boundaries, while periodic boundary conditions were applied on the left and right boundaries. The use of periodic boundary conditions on the left and

right hand boundaries allows a direct one-on-one comparison between the results for a single grid calculation and the patched grid calculation where the upper and lower regions were slightly displaced one from the other. For the rectangular domain considered, with the prescribed boundary conditions, the exact analytic solution for this test case is

$$T(x,y) = 1.0 + 0.02 \sin(2\pi x) \cosh(2\pi(z - 0.5)) \qquad 0 < x < 1$$
$$0 < z < 1$$

In this problem, the interpolation strategy used was the simplest type - the dependent variable on the patched line varies linearly within an engulfed cell.

This test case was chosen in order to study the feasibility of the algorithm for a problem governed by a single partial differential equation prior to considering the problem of flow over two tandem airfoils, which is governed by a coupled set of partial differential equations. It further offers the benefit of allowing comparison of the numerical solution to the exact analytic solution, thereby assessing the accuracy of the procedure. Also, it is cost effective in debugging a new numerical procedure by using a simple problem.

Figure 5 shows the temperature contours of the calculations in the patched grid configuration using the noniterative patched algorithm. Figure 6 shows the temperature contours of the calculations using the single grid. Note the displacement in the patched grid between the upper and lower regions. Results from both calculations are almost identical, which indicates that the accuracy of the procedure using the linear interpolation strategy is adequate. The developed technique was further tested in a nonuniform patched grid configuration. Excellent agreement was again obtained.

Test Case 2 - Uniform Flow

The second step to validate the developed algorithm is to perform a calculation closely related to fluid flow problems but yet having a known analytical solution. A sample calculation was performed to verify the algorithm by considering the case of an incompressible uniform laminar flow. The Navier-Stokes equations were solved in a rectangular domain in which periodic conditions were specified in the transverse direction. The initial conditions have streamwise variation in the dependent flow variables.

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Obviously, the converged solution for this case is uniform flow with no
streamwise variation. At the center of the rectangular domain a patch boundary
was specified, i.e. the coordinate system on each side of the boundary was
displaced relative to one another. The periodic conditions were employed here
to simulate the actual boundary conditions required for a cascade flow field.
At the upstream boundary the stagnation pressure was prescribed and at the
downstream boundary the static pressure was set and kept constant. In light of
the periodic conditions and the upstream and downstream boundary conditions,
the velocity field at steady state should be uniform. A converged steady state
solution was obtained with the expected behavior. This demonstrates the
operation of the developed patched grid technique for the coupled set of
Navier-Stokes equations. During this stage of study, a band solver for a block
banded matrix, which is required for a system of equations when applied to the
patched grid, was also implemented.

Test Case 3 - Tandem Joukowski Airfoils

For the final case, the developed patched grid algorithm was applied to
fluid flow calculations for a cascade of displaced Joukowski airfoils. Since
linear interpolation strategy seemed to be quite adequate in the previous case
studies, an obvious step is to use the linear interpolation strategy for the
tandem Joukowski airfoils and evaluate the adequacy of the linear interpolation
in these cases.

Two calculations (3a and 3b) were performed: In the first case, the
second airfoil was displaced by 0.002 chord while in the second case, it was
displaced by 0.525 chord. In both cases, the axial spacing between the two
airfoils was 0.75 chord and the stacking distance was 1.05 chord. Figures 7
and 8 show the coordinate system for the case 3a and case 3b calculations,
respectively. In the first case, the vertical displacement is very small
(about 0.2% chord length), therefore, the resulting flow fields are expected to
be the same as those using the continuous deformed grids. Figures 9, 10, 11
and 12 show the streamwise velocity and pressure contours for this case. In
comparing Figures 9 and 10 with 11 and 12, which show results for the single
grid calculation, the contour levels are almost identical. This indicates that
the patched grid algorithm works very well and has the same accuracy as the
continuous deformed grid calculations in a limiting situation. In this study
it was also found that the patched grid procedure required the same number of

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steps to converge (to steady state) to the same accuracy in the continuous grid calculations. However, it is worth noting that there is a slight oscillation in the pressure field near the patch lines. Figure 13 shows the pressure coefficient along the top surface of the second airfoil. In comparing Figures 13 and 14, they appear almost identical. This indicates that the slight numerical oscillation found upstream (ahead of the second airfoil) does not have adverse effects on the surface pressure of the second airfoil. Removal of this pressure oscillation using a higher order interpolation scheme will be discussed later.

In the case 3b calculation, the second airfoil is displaced a distance of half of the passage width (stacking distance). For this case, symmetric flow should be expected. Figures 15 and 16 show the streamwise velocity and pressure for this calculation. The convergence rate was approximately the same as for the slightly displaced case discussed above. Both the velocity and pressure show symmetric results as expected. Of particular interest are the contours representing the wake as they pass through the patch line. As can be seen, the transition across the wake of the velocity contours are very smooth. In regard to pressure contours, some small oscillations are noted. It is believed that this results from the use of a linear interpolation formula at the patch line, which may lead to an inconsistency or a loss in accuracy. In order to support this hypothesis, calculations for case 3a were repeated using a quadratic interpolation scheme. It should be noted that the technology developed in this study is not limited to linear interpolation; replacement of the linear interpolation scheme with a higher order scheme is a relatively straightforward process. A preliminary study (from results of case 3a) shows that the quadratic interpolation scheme does improve the accuracy of the solutions near the patched line and the small pressure oscillation found when using a linear interpolation scheme disappeared.

In regard to run time, the present patched grid approach with linear interpolation requires approximately 1.0 seconds per time step for a grid of 145 × 45 points. This represents an increase of approximately 80 percent over a single grid calculation. Based upon operation count, quadratic interpolation is expected to require an extra 10 percent time. It should be noted that the present implicit patched grid approach required less time than an explicit patched grid approach with one iteration per time or relaxation step. Since, in general, it is expected that at least four or five iterations per time step

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would be required to obtain rapid convergence or to follow transient solutions with time steps limited by physical considerations, the present implicit patched grid approach would be advantageous over other possible procedures.

CONCLUSIONS

In summary, when compared to the single continuous grid, the new noniterative patched grid requires the same number of time steps to converge and less than twice the computer time per time step. Therefore, although there is a run time penalty, it is not severe and the patched grid has very definite advantages in maintaining near orthogonality and maintaining high resolution in regions where large flow gradients are expected. When compared to the iterative patched approach, it is expected that for steady state problems the iterative patched grid approach would require at least three or four iterations per time step if single grid convergence rates were to be maintained, thereby incurring a significant run time penalty when compared to the present approach. Similarly, for unsteady flows several iterations per time step would be required unless the time step were made very small. Even with several iterations per time step, the present approach may allow larger time steps since time step size in the present approach is limited by flow physics rather than the explicit boundary conditions associated with an iterative approach. Thus, the procedure developed in Phase I appears to combine the flexibility of the iterative patched grid approach while not suffering the same severe run time penalty when compared with the single continuous grid.

RECOMMENDATIONS

Since the present Phase I effort has successfully demonstrated the efficiency of the proposed procedure, the next goal is to extend, generalize and apply the developed efficient noniterative patched grid algorithm to perform Navier-Stokes analysis for steady or time-dependent flows for three-dimensional multi-body configurations. The proposed three-dimensional algorithm, once developed, can be applied to various practical problems, e.g., three-dimensional rotor-stator configuration in an axial compressor or turbine -- steady and unsteady state studies, in particular, rotor-stator with unequal pitch, stator-rotor-stator situation, compressor-diffuser combination in a

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radial turbomachine with guide vanes; helicopter-fuselage rotor interaction; etc. The generalized noniterative patched grid algorithm should make more tractable a large variety of paractical problems which currently pose considerable difficulty to a Navier-Stokes approach due to the complexity of the geometric shapes involved.

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4. Weinberg, B.C., Yang, R.-J., McDonald, H. and Shamroth, S.J.: Calculation of Two- and Three-Dimensional Transonic Cascade Flow Fields Using the Navier-Stokes Equations, Journal of Eng. for Gas Turbines and Power, Vol. 108, pp.93-102, 1986.

5. Briley, W.R. and McDonald, H.: Solution of the Multidimensional Compressible Navier-Stokes Equations by a Generalized Implicit Method, Journal of Comp. Physics, Vol. 24, pp. 372-397, August 1977.

6. Briley, W.R. and McDonald, H.: On the Structure and Use of Linearized Block Implicit Schemes, Journal of Comp. Physics, Vol. 34, No. 1, pp. 54-72, January 1980.
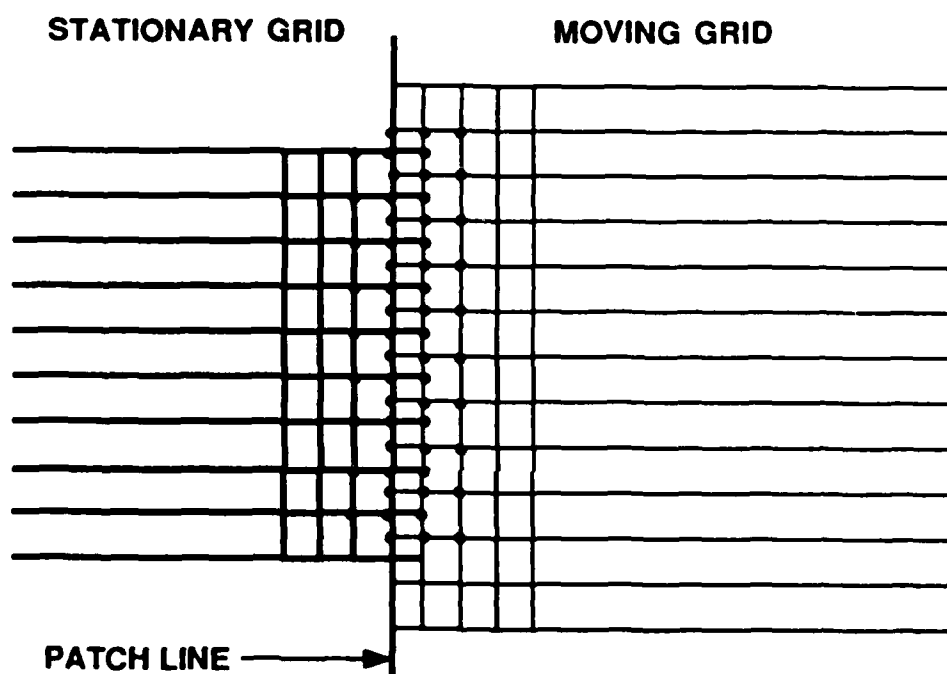
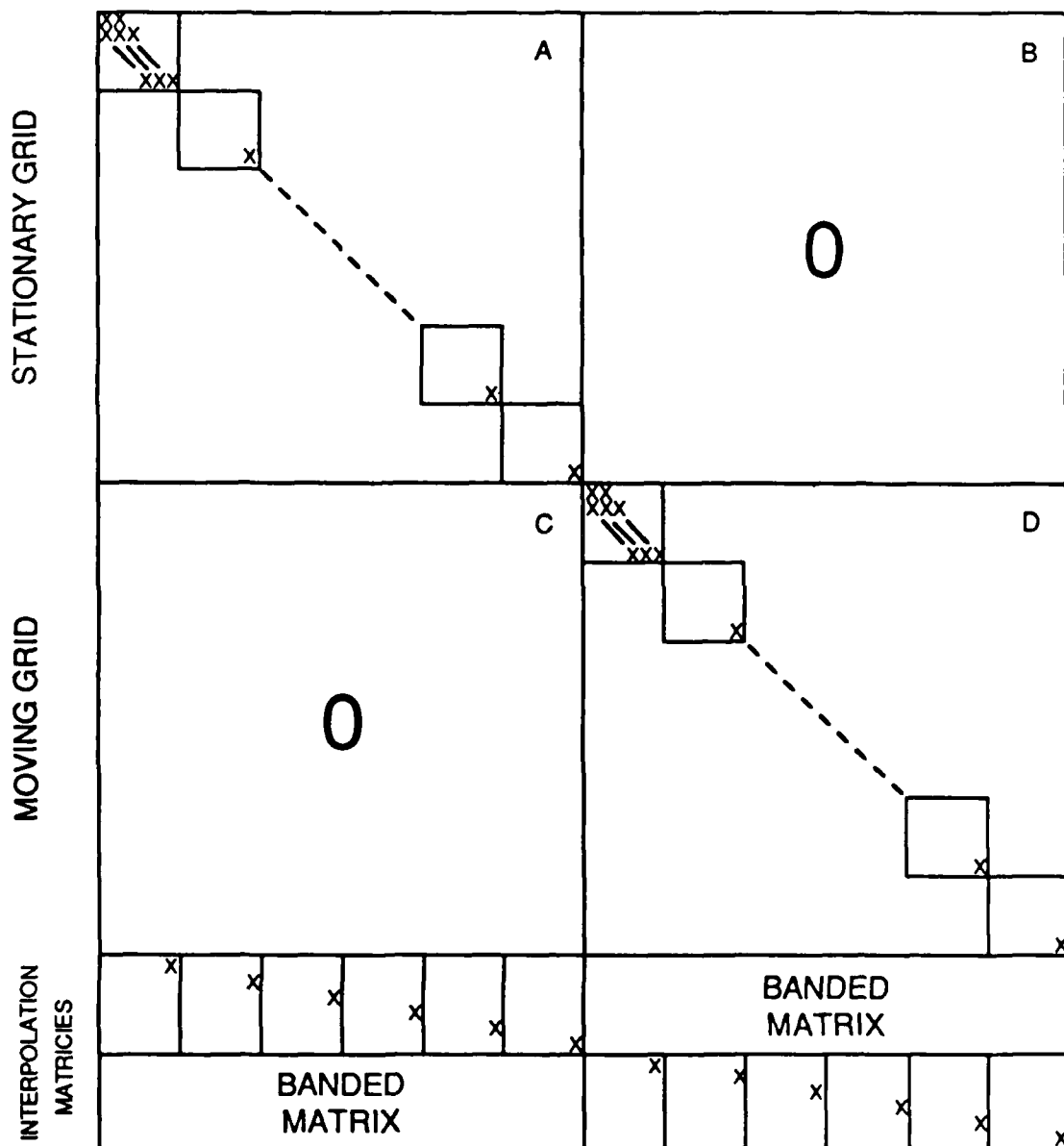Figure 1.   Schematic of the Patched Grid.

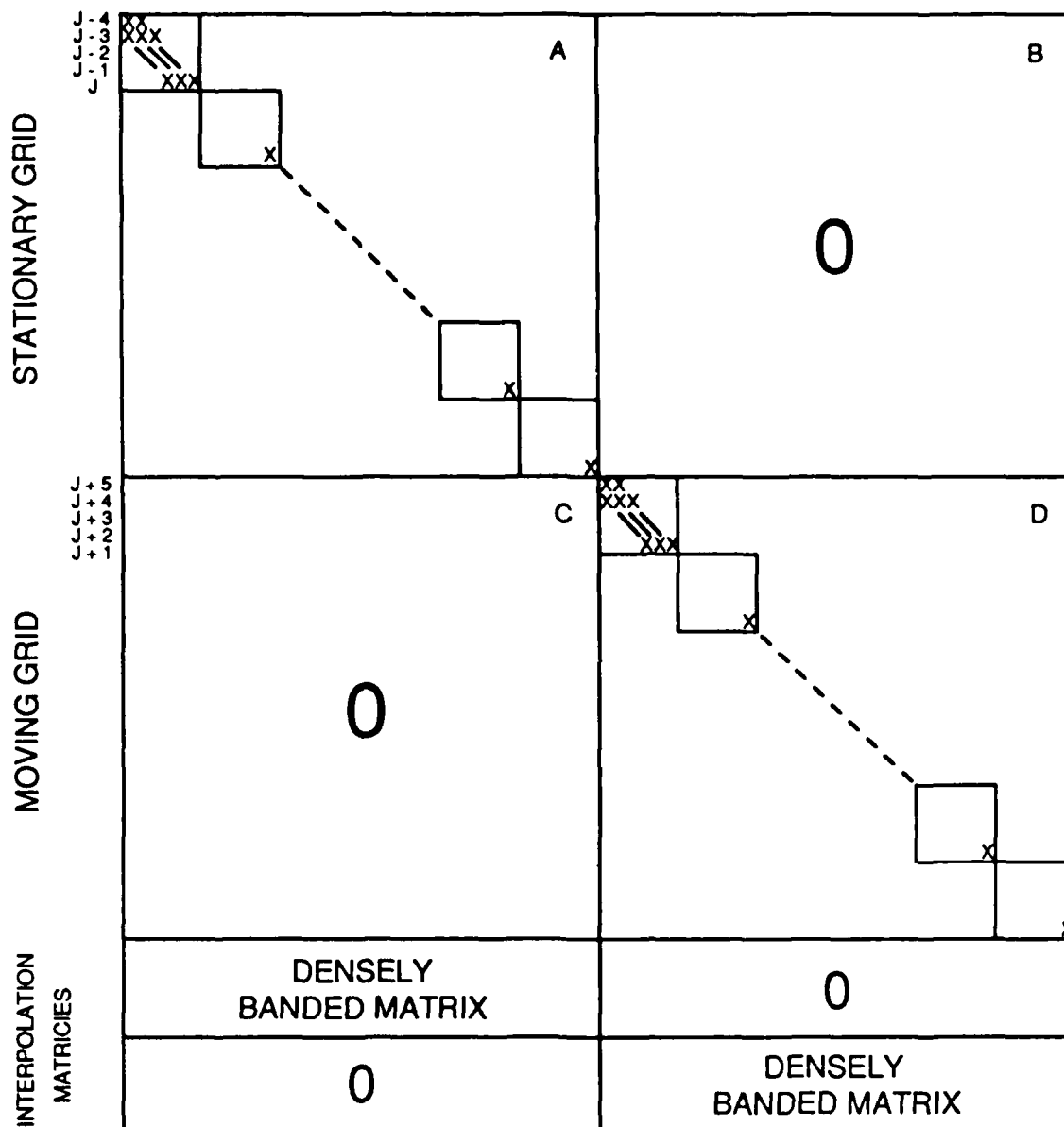Figure 2. Matrix Structure for 2-D Noniterative Solution Procedure.

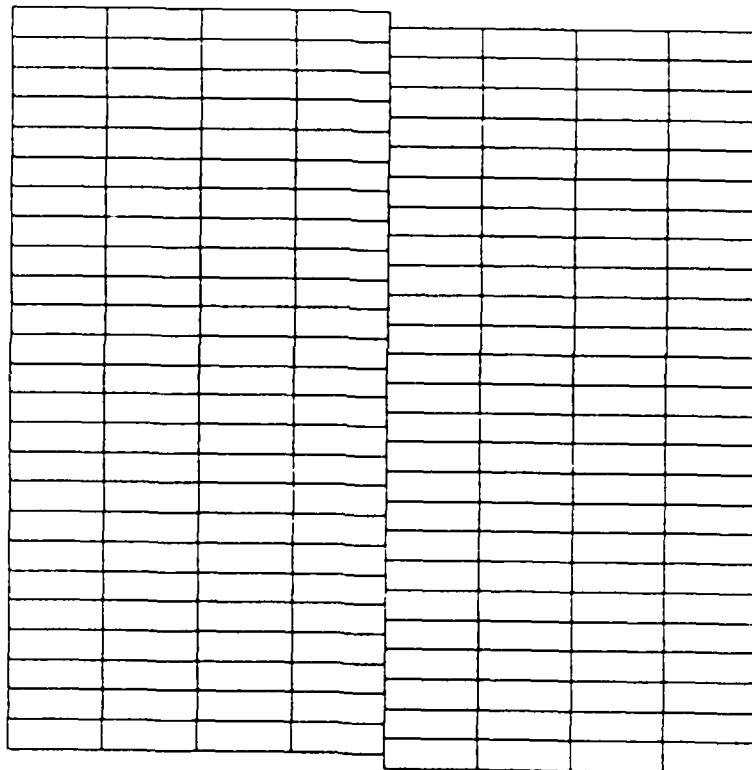Figure 3. Modified Matrix Structure for Noniterative Solution.

PATCHED SOLUTION

GRID

GRID

26x11

Figure 4

-17-
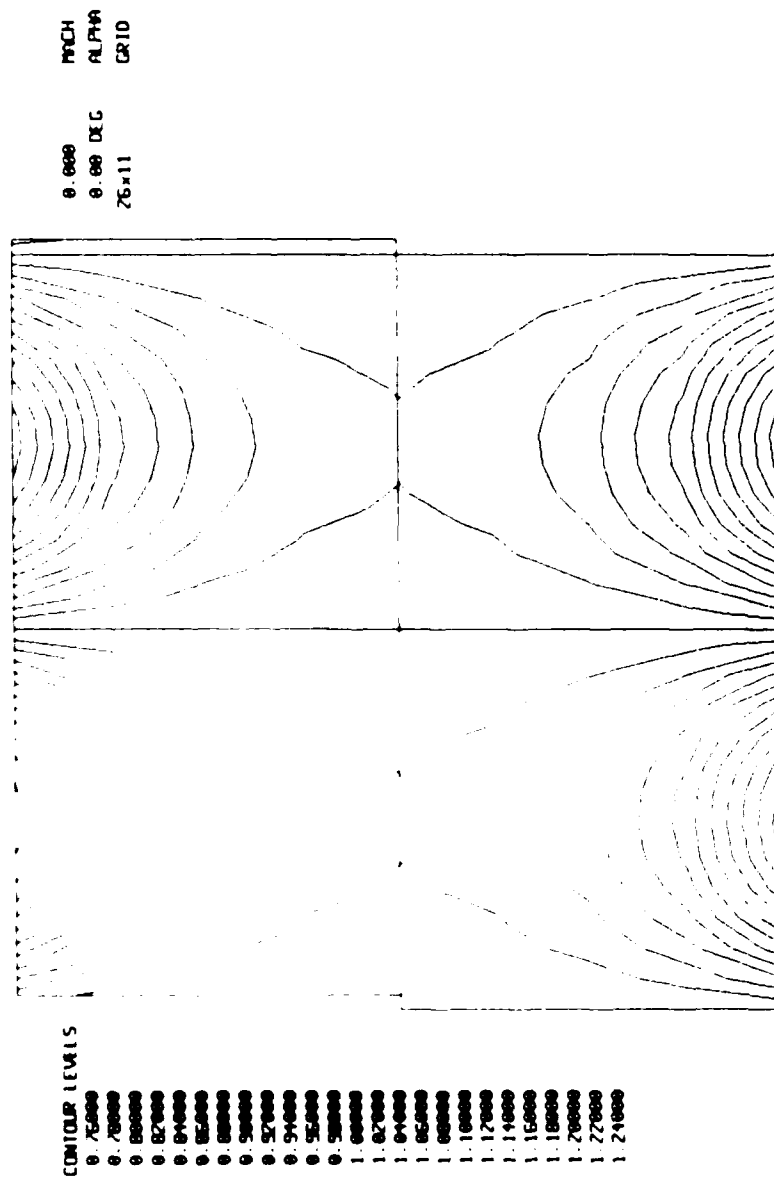
STAGNATION ENERGY PER UNIT VOLUME
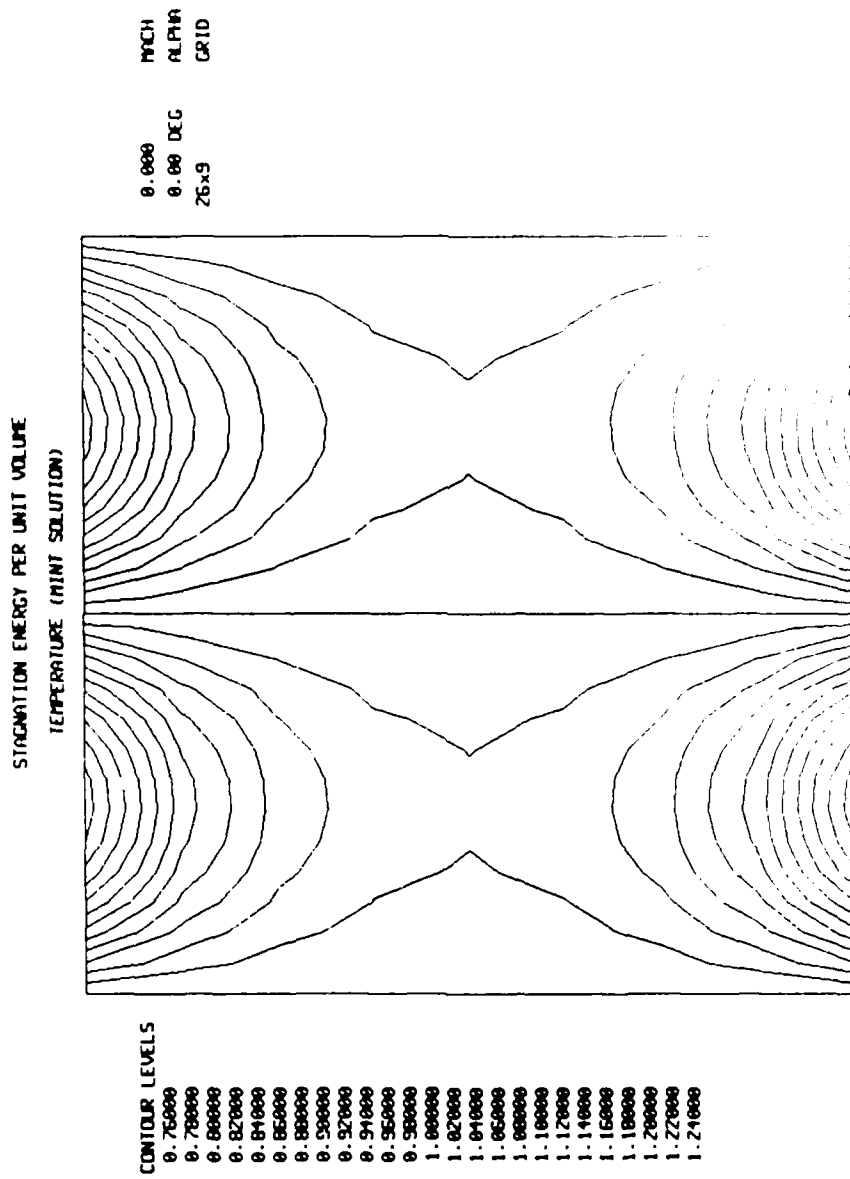
TEMPERATURE (PATCHED GRID)

MACH   0.000

ALPHA   0.00 DEG

GRID   76×11

CONTOUR LEVELS
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0.80000
0.82000
0.84000
0.86000
0.90000
0.92000
0.94000
0.96000
0.98000
1.02000
1.04000
1.06000
1.10000
1.12000
1.14000
1.16000
1.18000
1.20000
1.22000
1.24000

Figure 5

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STAGNATION ENERGY PER UNIT VOLUME

TEMPERATURE (MINT SOLUTION)

MACH   0.000
ALPHA   0.00 DEG
GRID   76x9

CONTOUR LEVELS
0.76000
0.78000
0.80000
0.82000
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0.86000
0.88000
0.90000
0.92000
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0.96000
0.98000
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1.02000
1.04000
1.06000
1.08000
1.10000
1.12000
1.14000
1.16000
1.18000
1.20000
1.22000
1.24000

Figure 6

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GRID

TANDEM DISPLACED JOUKOWSKI AIRFOILS (0.002)

PATCHED GRID (LINEAR)

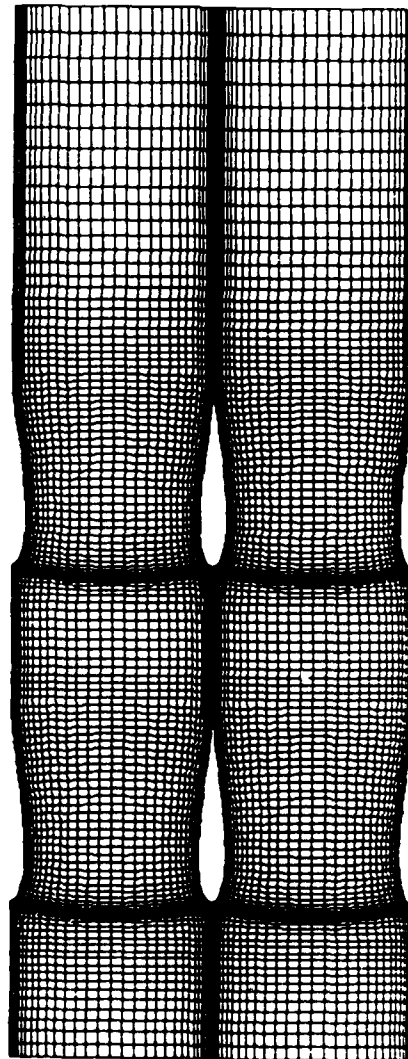144x45   GRID 1
144x45   GRID 2

Figure 7

GRID
TANDEM DISPLACED JOUKOWSKI AIRFOILS (0.525)
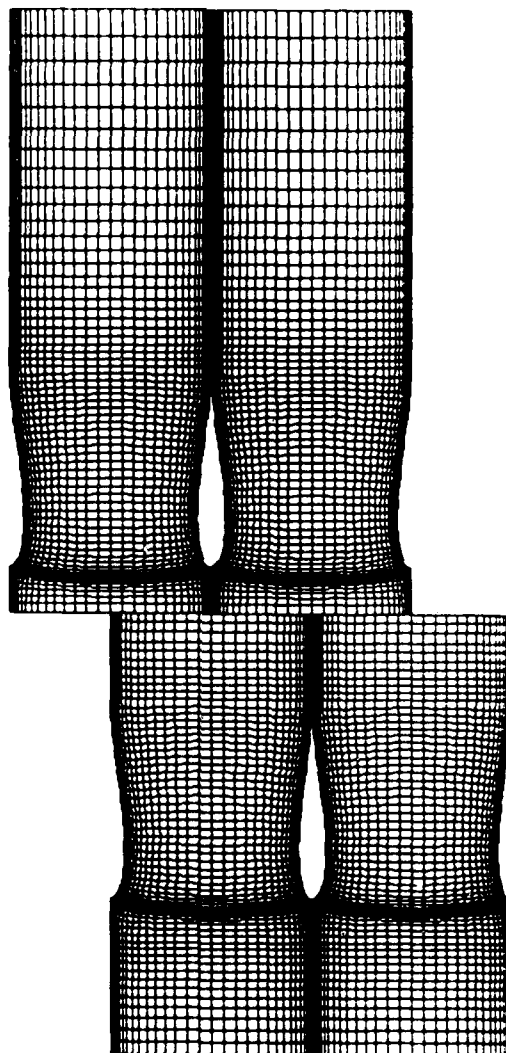PATCHED GRID (LINEAR)

144x45   GRID 1
144x45   GRID 2

Figure 8

U VELOCITY
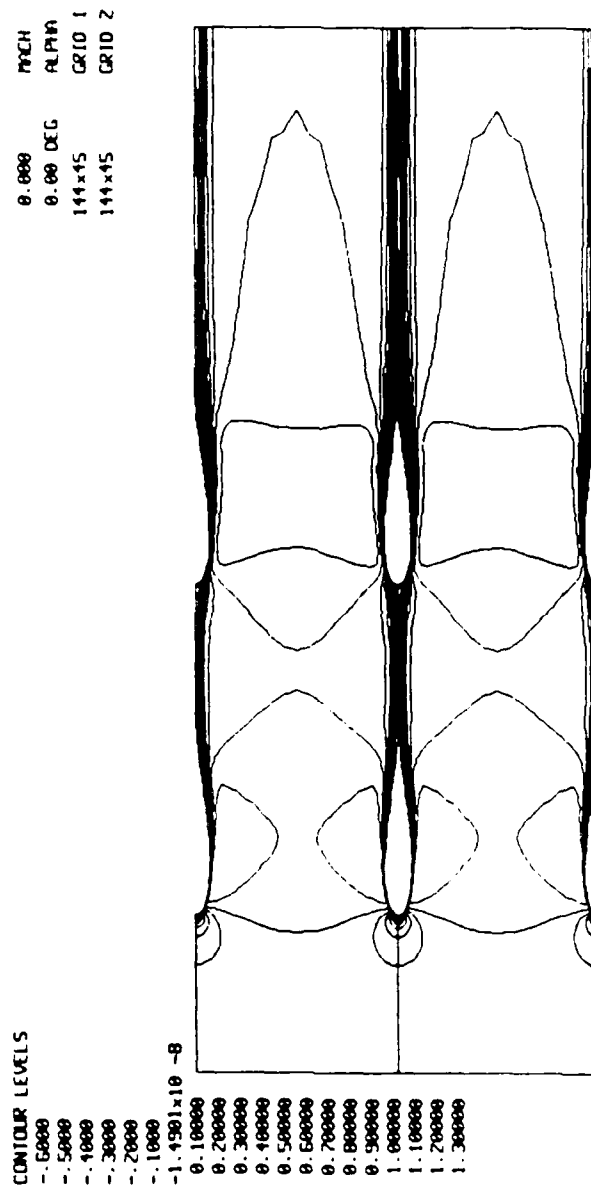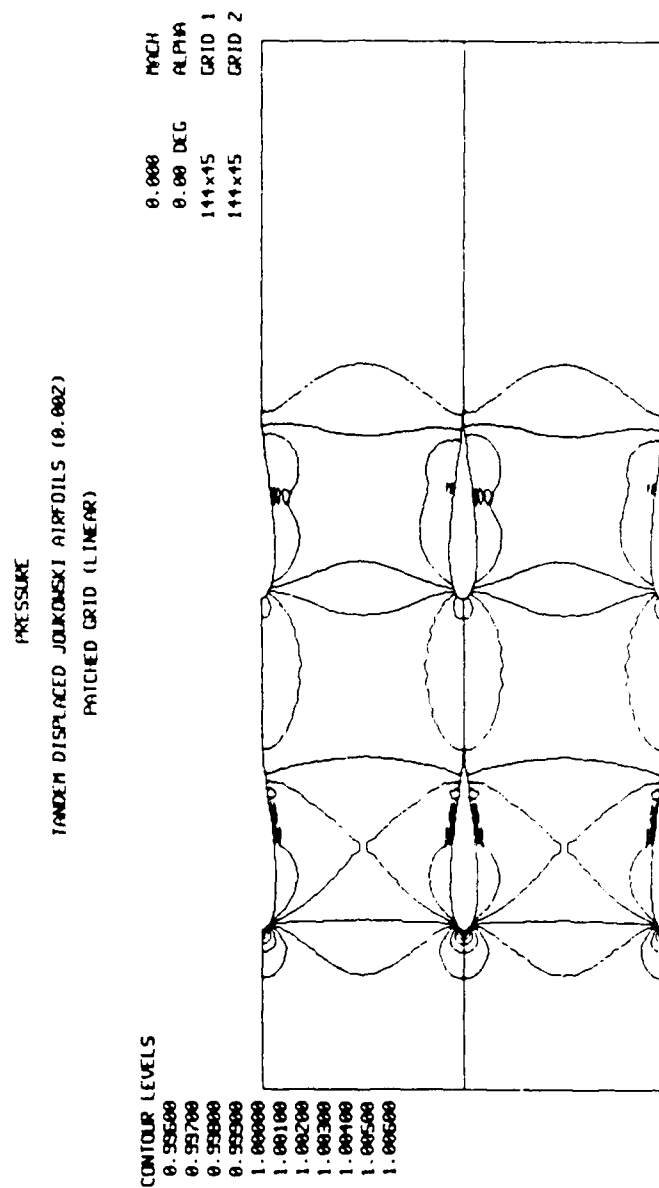TANDEM DISPLACED JOUKOWSKI AIRFOILS (0.002)
PATCHED GRID (LINEAR)

MACH   0.000
ALPHA   0.00 DEG
GRID 1   144x45
GRID 2   144x45

CONTOUR LEVELS
-.6000
-.5000
-.4000
-.3000
-.2000
-.1000
-1.4501x10 -8
0.10000
0.20000
0.30000
0.40000
0.50000
0.60000
0.70000
0.80000
0.90000
1.00000
1.10000
1.20000
1.30000

Figure 9

-22-

PRESSURE

TANDEM DISPLACED JOUKOWSKI AIRFOILS (0.002)

PATCHED GRID (LINEAR)

CONTOUR LEVELS
0.99500
0.99700
0.99800
0.99900
1.00000
1.00100
1.00200
1.00300
1.00400
1.00500
1.00600

MACH	0.000
ALPHA	0.00 DEG
GRID 1	111x45
GRID 2	111x45

Figure 10

U VELOCITY

TANDEM DISPLACED JOUKOWSKI AIRFOILS (0.002)

CONTINUOUS DEFORMED GRID

MACH	0.000	
ALPHA	0.00 DEG	
GRID 1	145x45	
GRID 2	145x45	

CONTOUR LEVELS

-.6000
-.5000
-.4000
-.3000
-.2000
-.1000
-1.1901x10 -8
0.10000
0.20000
0.30000
0.40000
0.50000
0.60000
0.70000
0.80000
0.90000
1.00000
1.10000
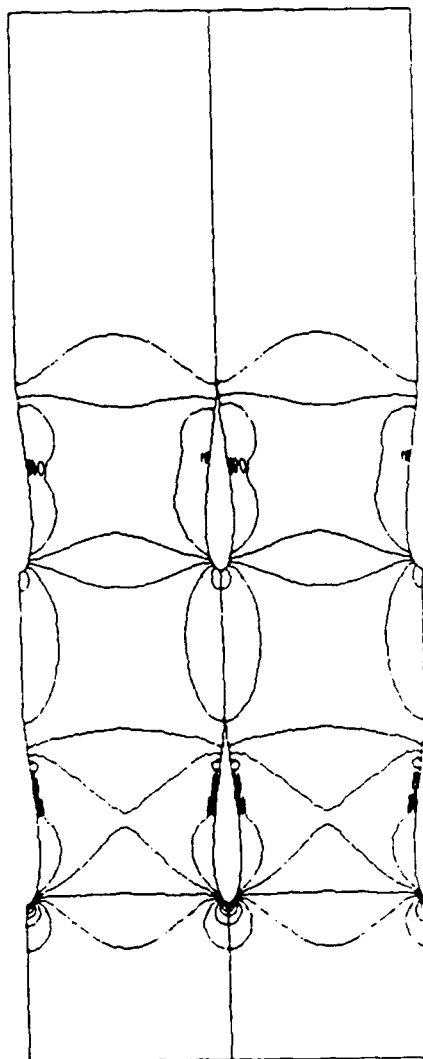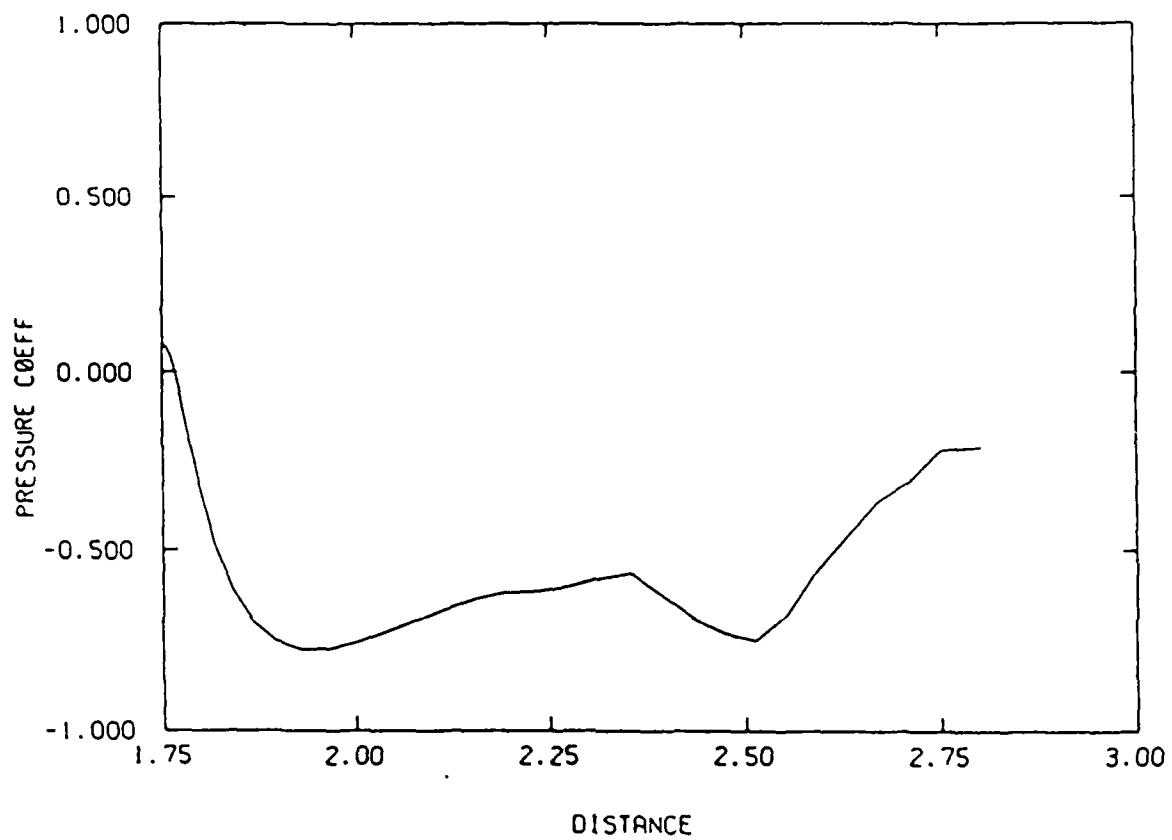1.20000
1.30000

Figure 11

Figure 12

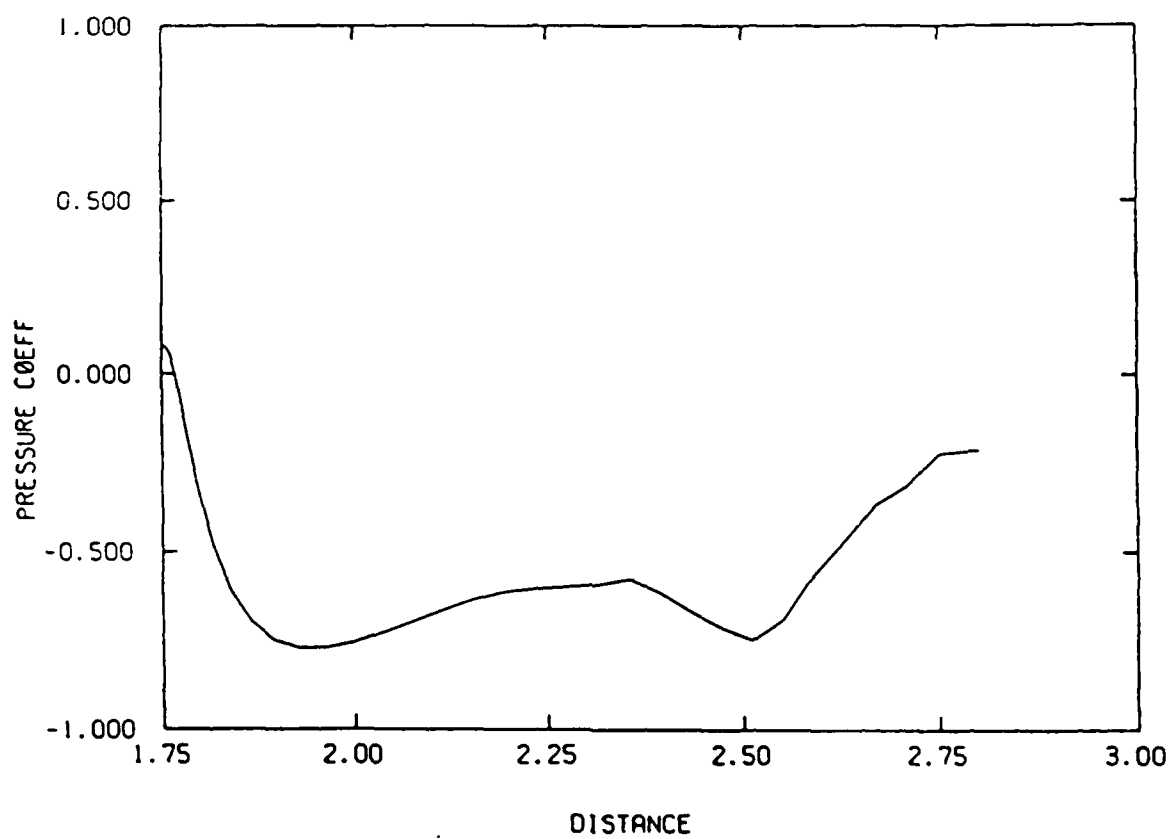Figure 13. Pressure Coefficient on Top Surface of Airfoils
Continuous Grid (0.002).

Figure 14. Pressure Coefficient on Top Surface of Airfoils.
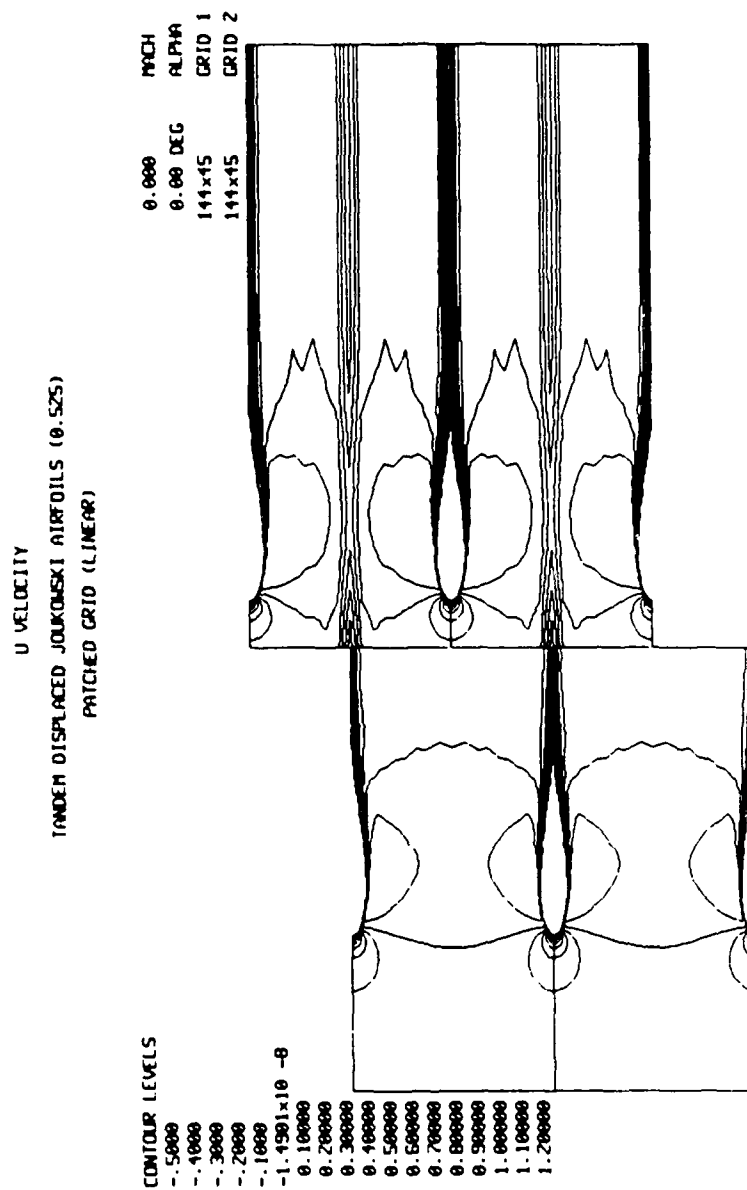Patched Grid (0.002).

U VELOCITY

TANDEM DISPLACED JOUKOWSKI AIRFOILS (0.525)

PATCHED GRID (LINEAR)

MACH   0.000
ALPHA   0.00 DEG
GRID 1   141×15
GRID 2   141×15

CONTOUR LEVELS
-.5000
-.4000
-.3000
-.2000
-.1000
-1.1901×10 -8
0.10000
0.20000
0.30000
0.40000
0.50000
0.60000
0.70000
0.80000
0.90000
1.00000
1.10000
1.20000

Figure 15

PRESSURE

TANDEM DISPLACED JOUKOWSKI AIRFOILS (0.525)

PATCHED GRID (LINEAR)

MACH 0.000
ALPHA 0.00 DEG
GRID 1 144x45
GRID 2 144x45

CONTOUR LEVELS
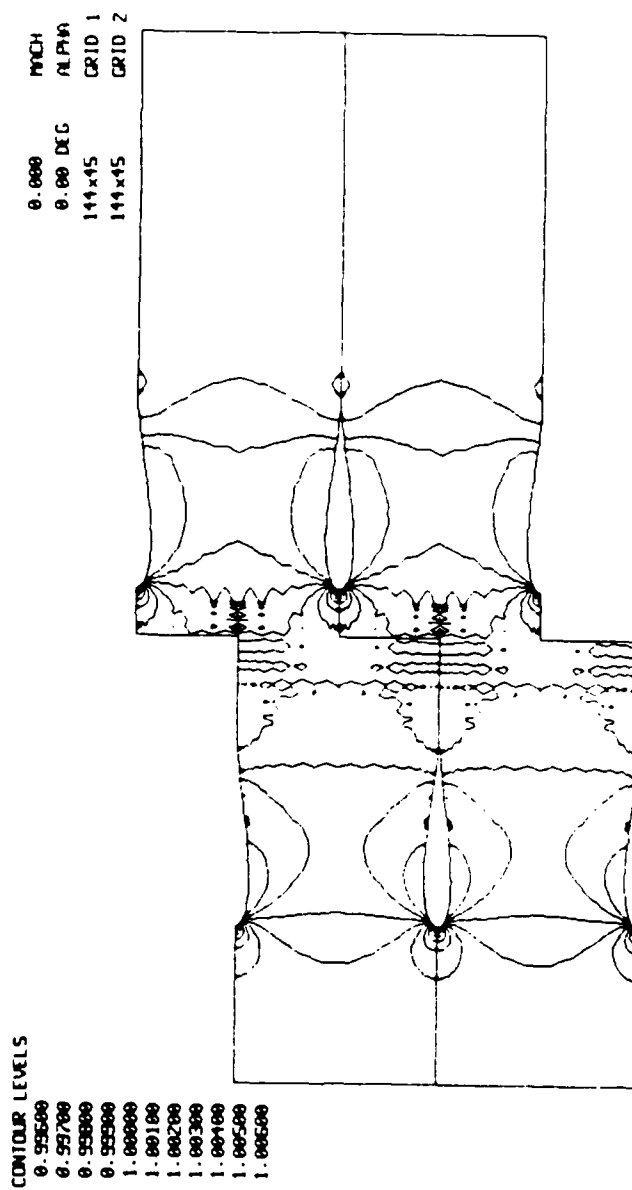0.99500
0.99700
0.99800
0.99900
1.00000
1.00100
1.00200
1.00300
1.00400
1.00500
1.00600

Figure 16

END

DATE

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